AC phase-controlled bridges – ground and main properties

Surdu Michael¹, Surdu Dmytro²

¹, ² Institute of space investigation of NANU, Ukraine
E-mail: Michaelsurdu1941@gmail.com

Abstract

This work describes the new approach to the improving of the accuracy of the bridge impedance measurements. By this approach, the bridge balance in a wide impedance range uses the control of the phases of the aggregate of the cardinal signals only. The optimal structure and algorithm of the control of this aggregate of the cardinal signals is determined. Using this approach accurate impedance meters with low size and price are developed. In the limit, this approach gets us the possibility to develop fully integrated impedance meters. Report shortly describes universal impedance meters with phase control, their balance and calibration algorithms.

Keywords: measurement, impedance, phase control, uncertainty, transfer coefficient, calibration.

1. Introduction

Impedance measurement has the long history [1]. However, to this day, the creation of precise equipment with high accuracy, small dimensions and weight, does not cease to be relevant.

To determine the parameters of the impedance, many measurement methods have been proposed. Bridge methods get the highest accuracy of impedance measurement.

Historically, four-arm AC bridges were the first used for accurate impedance measurements [1, 2, 3]. In these bridges, to achieve balance, the indicator compares two output voltages of two dividers created by the reference standards and the object to be measured. The balancing of bridges is carried out by adjusting one of these voltages using reference resistance and/or capacitance boxes.

In the transformer bridges that appeared later [1, 4, 5, 6], two voltages applied to the impedance standard and the measurement object are created using inductive dividers. To balance the bridge, one of the mentioned voltages is adjusted by switching the number of turns of the inductive divider. Such bridges provide measurement with high accuracy, but they are cumbersome and have a relatively narrow frequency range.

Since the late 70s, the accurate digital-to-analog converters (DAC) have been developed. On this base, AC generators were developed [7, 8]. In these generators the DC voltage acts as an input signal, and the output signal is a piecewise approximated sinusoidal voltage (abbreviated – DDS) [8, 9]. DDS were used to create various AC bridges [9, 10]. In these bridges, amplitude and phase of the balancing signal were directly adjusted using DAC. Accuracy of these bridges isn’t too much.

In the mid-90s, intensive works were devoted to creation of AC voltage standards based on the Josephson effect [11]. These standards are precision DDS based on either a piecewise linear approximation of a sine waveform or a pulse-driven Josephson array. These works, in the last decade, were used to develop AC bridges [12, 13]. Here two DDS based on the Josephson effect act as two arms of the bridge. The standards being compared act as another two bridge arms. To balance such bridges, the amplitude and phase of these DDS are adjusted with high accuracy. Such bridges are potentially very accurate (if the influence of other sources of error inherent in impedance measurement is completely eliminated). However, their complexity, due to superconductivity, is great.

All these AC bridges use for balance adjustment of the amplitude and phase of the proper voltage sources...
2. Optimization of the system of cardinal signals

Let us consider the possibility of changing the signal amplitude by changing only the phases of a set of signals.

It is clear that the amplitude of a single signal only can be changed by changing its frequency \( \omega \) or phase \( \varphi \). Therefore, consider the sinusoidal voltage \( U_j \), which is the sum of \( k \) separate coherent sinusoidal voltages \( U_i \) of the same frequency \( \omega \) with the amplitude \( U_{mj} \) and phase \( \varphi_i \):

\[
U_c = \sum_{j=1}^{k} U_j = \sum_{j=1}^{k} U_{mj} \sin(\omega t + \varphi_i).
\] (1)

To answer on these questions let us consider two cases:

- when the number of voltage sources is even;
- when the number of voltage sources is odd.

Let the number \( k \) of voltage sources in equation (1) be even. We'll divide the total sum of \( k \) voltage sources by \( \frac{k}{2} \) pairs. Then equation (1) takes the form:

\[
U_c = \sum_{j=1}^{\frac{k}{2}} U_{mj} \sin(\omega t + \varphi_{1j}) = \sum_{j=1}^{\frac{k}{2}} \left[ U_{m1j} \sin(\omega t + \varphi_{1j}) + U_{m2j} \sin(\omega t + \varphi_{2j}) \right],
\]

where

\[
U_{m1j}, U_{m2j}, \varphi_{1j}, \varphi_{2j} - \text{amplitudes and phases of the } j\text{-th pair of voltages.}
\]

Formulas (3) determine the amplitude \( U_{mij} \) and phase \( \varphi_{ij} \) of the voltage \( U_{ij} \) [14]:

\[
U_{mij} = \sqrt{U_{m1j}^2 + U_{m2j}^2 + 2U_{m1j}U_{m2j} \cos(\varphi_{1j} - \varphi_{2j})};
\]

\[
tg\varphi_{ij} = \frac{U_{m1j} \sin \varphi_{1j} + U_{m2j} \sin \varphi_{2j}}{U_{m1j} \cos \varphi_{1j} + U_{m2j} \cos \varphi_{2j}}.
\] (3)

Firstly, we find the optimal ratio \( r = \frac{U_{m1j}}{U_{m2j}} \) of the amplitudes of the cardinal voltages and determine the algorithm of their phases \( \varphi_{1j} \) and \( \varphi_{2j} \) control \( (\varphi_{1j} - \varphi_{2j} = 2\Psi) \).

Taking into account the above, we transform equations (3) to the form:

\[
U_{mij} = U_{m1j} \sqrt{1 + r^2 + 2r \cos 2\Psi};
\] (4)

\[
tg\varphi_{ij} = \frac{\sin \varphi_{1j} + r \sin \varphi_{2j}}{\cos \varphi_{1j} + r \cos \varphi_{2j}}.
\]

In the literature, a voltage or current vector is often called a phasor. Here, for brevity, we will extend this term to a device that generates voltage or current according to equations (3), (4).

Equation (4) determines the dependence of the phasor voltage on the ratio \( r \) of the amplitudes of the cardinal signals and the difference \( 2\Psi \) in their phases.

2.1. The voltage system, described by the equation (1), does not let us provide an accurate AC bridge. It needs to be optimized. As the optimization criteria, we'll take the dynamic range of the relative change of the \( U_c \) and we'll require that this value reach a maximum.

In the process of system (1) optimization it is necessary to answer on the following questions:

- What should be the ratio of these voltages amplitudes?
- What is the optimal law of the voltages phase control?

The complexity of such a source determines the technical and economic adjectives of the bridge.

In this work, we propose a new method for adjusting the amplitude of the bridge balancing signal in a wide range of values. This method is based on the adjustment of the phases of the cardinal balancing signals only. Using this approach, we can achieve high accuracy, with small dimensions and weight of the impedance meter.
We impose two restrictions on these values:

1. The total voltage $U_{mcj}$ is a non-linear function of the phase difference $2\Psi_j$. Therefore, the voltage sensitivity $\Delta U_{mcj}$ before changes of the $U_{mcj}$ in the phase difference $2\Psi_j$ is not constant in the dynamic range of phases. Let’s define the value $\Delta U_{mcj}$:

$$
\Delta U_{mcj} = \left(U_{mcj} \sqrt{1 + r^2 + 2r\cos 2\Psi_j}\right) \Delta \Psi = -U_{m1j}r \frac{\sin 2\Psi_j}{\sqrt{1 + r^2 + 2r\cos 2\Psi_j}} \Delta \Psi.
$$

(5)

From (5) we see that the voltage $U_{mcj}$ sensitivity $\Delta U_{mcj}$ changes in the control range of the phases $\Psi_j$ according to an approximately sinusoidal law from a relative value of 1 to zero. Obviously, it is impossible to balance the bridge with zero sensitivity. Therefore, we introduce a restriction on the range of angle $\Psi_j$ change: $\frac{\pi}{4} \leq \Psi_j \leq \frac{3\pi}{4}$. Then the control range insignificantly, in $\sqrt{2}$, decreases, and variations in the bridge sensitivity in the control range do not exceed $\sqrt{2}$. Such changes in sensitivity no longer complicate the balancing of the bridge.

2. The voltages $U_{mcj}$ and $U_{m1j}$ are generated by DDS, which have a limited voltage range $U_{max}$. Therefore, all generated voltages must obey the restrictions:

$U_{mcj} \leq U_{max}$; $U_{m1j} \leq U_{max}$; $U_{m2j} \leq U_{max}$ i.e. all arguments in (3), (4) must be finite and $0 \leq r \leq r_{max}$.

From (4), we find the dynamic range $D$ of the phasor, defined as the ratio of the maximum possible voltage $U_{mcj}$ value (at $2\Psi_j = \frac{\pi}{2}$) to its minimum value (at $2\Psi_j = \pi$):

$$
D = \frac{U_{mcj}}{U_{mcj}}_{max} = \left(\frac{U_{mcj}}{U_{mcj}}_{min}\right)
$$

(6)

$$
= \pm \sqrt{1 + r^2 + 2r\cos \frac{\pi}{2}} \pm \sqrt{1 + r^2 - 2r\cos \pi} = \pm \sqrt{1 + r^2 - 2r}.
$$

From (6) we see that the dynamic range $D$ of the phasor tends to its maximum value (infinity) as $r$ tends to 1. Therefore, we assume that:

$$
r = 1 \text{ always.}
$$

(7)

This equality does not guarantee that the change in the phases of the cardinal voltages will change the amplitude of the output voltage of the phasor only. To determine the law of the phases of cardinal voltages control, we equate to zero the phase of the output voltage of the phasor, which should not change when the phases of the cardinal voltages change, i.e.:

$$
tg \varphi_{cj} = \frac{\sin \Psi_{1j} + r \sin \Psi_{2j}}{\cos \Psi_{1j} + r \cos \Psi_{2j}} = 0,
$$

(8)

where:

$$
\Psi_{1j}, \Psi_{2j} - \text{angles between cardinal voltages } U_{m1j}, U_{m2j} \text{ and voltage } U_{mcj}.
$$

From (8) with $r = 1$ we obtain that:

$$
\Psi_{1j} = -\Psi_{2j} \text{ always.}
$$

(9)

Under conditions (7) and (9), the laws of control of the amplitude and phase of the total voltage take the form:

$$
U_{mcj} = 2U_{m1j} \cos 2\Psi_j, \Delta \varphi_{cj} = \Delta \varphi_{1j} = \Delta \varphi_{2j}.
$$

Fig. 1 demonstrates how the antiphase increment of the phases $\Psi_{11}$ and $\Psi_{21}$ of the cardinal vectors $U_{11}$ and $U_{21}$ to the $\Psi_{22}$ and $\Psi_{12}$ of the cardinal vectors $U_{12}$ and $U_{22}$ changes the amplitude of the summary vector $U_j$ from $U_{c1}$ to $U_{c2}$ and the sin-phase increment of these angles rotates the acting vector by an angle $\varphi_2 - \varphi_1$.

**Fig. 1. Vector diagram of the phase control**

Thus, by adjusting only the phases of two cardinal voltages, it is possible to vary within wide limits separately the amplitude and phase of the total voltage used to balance the bridge.

Let now that the phasor contains $\frac{k}{2}$ pairs of DDS, their output voltages are summed up, and the phases of the voltages of each pair are controlled according to the law (10). Then the equations $U_{mcj} = kU_{m1j} \cos 2\Psi_j$, $\Delta \varphi_{cj} = \Delta \varphi_{1j} = \Delta \varphi_{2j}$ describe law of control of the
phasor output voltage, i.e. the amplitude of the output voltage of the phasor increases, in relation to the DDS voltage amplitude, by a factor of \( k \), while maintaining the phase control law.

Let the number \( k \) of voltage sources in equation (I) will be odd.

We can select among all \( k \) voltage sources such \( n \) sources that \( n = k - 1 \). Then the sum of \( k \) voltages can be represented in the following form:

\[
U_c = \sum_{j=1}^{n} (U_{1j} + U_{2j}) + U_k. \tag{11}
\]

The first term on the right-hand side of (11) is the sum of the cardinal voltages pairs. The optimization of these pairs we perform as described above. Let us assume that the amplitude and phase of the voltage \( U_k \) are constant in time. Then we can consider this voltage as an additive component and easily take it into account in the phasor control algorithm.

2.2. Equation (6) shows that at \( r \to 1 \) the width of the dynamic range of the phasor tends to infinity, since in this case the minimum value of the total voltage \( U_r \) tends to zero. In practice, however, there are several factors that limit the minimum value of \( U_r \):

1. Not ideal equality of the cardinal voltages: \( U_{1j} \neq U_{2j} \);
2. Non zero discreteness of the DDS;
3. Non zero level of the noise at the output of the phasor.

Inequality of the cardinal voltages \( U_{1j} \neq U_{2j} \) leads to a zero shift of the phasor, which is taken into account as a result of calibration.

Phasor discreteness in the simplest case is determined by the ratio of the operating \( f_0 \) and clock \( f_d \) frequencies of the DDS and can be relatively large. However, we can reduce phasor discreteness to negligible values using multi-decade phasors. This question we will discuss below.

The noise of the DDS components determines the noise at the output of the phasor. This kind of interference can be reduced by the proper choice of components, but in principle, it cannot be reduced to zero.

We need to estimate how DDS noise converts into output noise of the phasor and limits its dynamic range. For this purpose, we assume that in each DDS, in addition to cardinal voltages, there are also some noise voltages \( \Delta U_{n1} \) and \( \Delta U_{n2} \). Taking into account equation (3), we can write:

\[
U_{mc} + \Delta U_{mc} \approx \sqrt{U_{mc}^2 + 2U_{mc}U_{n1} + U_{n1}^2} + (U_{n1} + U_{n2}) \cos 2\Psi,
\]

where:

\[
U_{mc}^2 = U_{m1}^2 + U_{m2}^2 + 2U_{m1}U_{m2} \cos 2\Psi; \quad 2\Psi = \Psi_2 - \Psi_1.
\]

From the last equation we find the following expression for the signal-to-noise ratio at the phasor output:

\[
\frac{\Delta U_{mc}}{U_{mc}} = \frac{(r + \cos 2\Psi) \Delta U_{n1} + (1 + r \cos 2\Psi) \Delta U_{n2}}{U_{m2}(1 + r^2 + 2r \cos 2\Psi)}. \tag{12}
\]

Let us assume that the noise \( \Delta U_{n1} \) and \( \Delta U_{n2} \) in both channels are not correlated and have the same average value. Usually it is true. Then, using the rules of addition of the random variables, from (12) we obtain:

\[
\frac{\Delta U_{n2}}{U_{m2}} \quad \text{and} \quad \frac{\Delta U_{mc}}{U_{mc}} \quad \text{are signal-to-noise ratio at the DDS and phasor outputs.}
\]

From expression (13) we determine how much the signal-to-noise ratio at the output of the phasor has changed in comparison with the signal-to-noise ratio of an individual DDS. Investigation of the equation (13) showed that the minimum ratio \( \frac{\Delta U_{mc}}{U_{mc}} \) is achieved at \( r = 1 \). This ratio is equal to:

\[
\frac{\Delta U_{mc}}{U_{mc}} = \frac{1}{\sqrt{2}} \frac{\Delta U_{n2}}{U_{m2}}. \tag{14}
\]

Formula (14) shows that at the output of the phasor the noise-to-signal ratio improves by a factor of \( \sqrt{2} \) compared to this ratio at the outputs of the DDS. If \( k \frac{f_0}{2} \) DDS pairs are used in the phasor, then the signal-to-noise ratio at the phasor output is improved by \( \sqrt{k} \) times. This is a very important property of the phasor.

3. AC bridge with phase-controlled balance

Fig. 2 shows the structure diagram of the AC bridge, balanced by the phasor \( \Phi \), with the output voltage \( U_c \) controlled in accordance with the laws (12).

Bridge contains synthesizer DDS, phasor \( \Phi \) with two synthesizers DDS, and DDS, MC control work of all DDS. Stable DC source \( U_{dc} \) supplies the inputs of these DDS. DDS generates the cardinal sinusoidal...
voltages $U_{11}$ and $U_{21}$, and reference voltage $U_0$. The amplitudes of these voltages have the same value. MC changes its phases in accordance with the algorithm described above. Adder $\Sigma$ folds the cardinal voltages $U_{11}$ and $U_{21}$ and applies resulting voltage $U_c$ to the measured impedance $Z_c$. Synthesizer DDS$_0$ creates the second arm of the bridge. Its output voltage $U_0$ supplies the impedance $Z_0$. Voltmeter VV measures the unbalanced voltage of the bridge at the junction of the impedances $Z_c$ and $Z_0$. MC sends the results of VV measurement to the PC. Last one processes them, balances the bridge and forms the measurement result.

4. Bridge balance procedure

To balance the bridge, we will change the phasor output voltage $U_c$ by algorithm described above and use the variation method [15]. In accordance with the variation method, we make the variation $\Delta U_{cv} = U_c \cos \Delta \Psi_c e^{-j\delta_c}$ of the voltage $U_c$, where: $\Delta \Psi_c$ - variational increment of angles $\Psi_1, \Psi_2$. VV measures the signal of the bridge unbalance before and after the variation. Next system of equations describes these measurements:

$$\frac{U_c - U_0}{Z_c} = I_n Z_{np} = a_n + j b_n;$$

$$\frac{U_{c0} + \Delta U_{c1}}{Z_c} - \frac{U_0}{Z_0} = I_n Z_{np} = a_c + j b_c,$$

where:

$I_n$ is the unbalance current of the bridge, $a_n$, $b_n$ and $a_c$, $b_c$ are the quadrature components of the VV output voltage before and after the variation, $Z_{np}$ is the VV cross impedance.

We can represent the initial phasor voltage $U_{c0}$ as the sum $U_{c0} = U_{cb} + \Delta U_{c0}$, where $U_{cb}$ is the voltage $U_c$ value at the bridge balance, $\Delta U_{c0}$ is the initial distance between the current state of the bridge and its balance point. Taking this into account, from (15) we find $\Delta U_{c0}$:

$$\frac{\Delta U_{c0}}{U_{c0}} = \sqrt{a^2 + b^2} \cdot \cos \Delta \Psi_c e^{-j(\delta_c + \Delta \delta_c)},$$

where:

$$a = a_n (a_c - a_n) + b_n (b_c - b_n);$$

$$b = b_n (a_c - a_n) - a_n (b_c - b_n);$$

$$\Delta \delta_c = \arctg \frac{b}{a}.$$

We balance the bridge introducing into the phasor the phase increments of the cardinal voltages, calculated according to (16). After that, the voltage $U_{c1}$, generated by the phasor, will be equal to:

$$U_{c1} = U_{c0} + \Delta U_{c0}.$$

This value of the phasor output voltage is used to calculate the result of the measurement according to the bridge balance equation.

The balance error of the bridge depends on the discreteness of the phasor, the VV discreteness and nonlinearity, noise and interference. Using the modern components, it is easy to construct a VV with a discreteness error of less than $10^{-5}$ and a nonlinearity of less than $10^{-4}$. Then, without taking into account noise, the error $\delta_0$ of the bridge balance don’t exceed $2 \cdot 10^{-4}$. The residual bridge unbalance is $\Delta U_{c1}$.

At the second stage of the bridge balance, we reduce the relative value of the variation to a value close to the error $\delta_0$ and, by the same ratio, increase the VV sensitivity. After that, we determine the bridge unbalance $\Delta U_{c1}$. The resulting value $\Delta U_{c1}$ we can use in two ways:

- if the discreteness of the phasor control is less than the permissible error of the bridge balance, we enter the control code $U_{c2} = U_{c1} + \Delta U_{c1}$ to the phasor and finish the bridge balance;
- if the discreteness of the phasor control is large, we perform a simple algebraic summation of the voltages $U_{c1}$ and $\Delta U_{c1}$. Using obtained voltage value $U_{c2}$, we calculate the measurement result.

Excluding noise and interference, the same VV parameters determine the unbalance error $\delta_3$ on the second stage. Therefore, the total error $\delta_3$ of
the bridge balance can be estimated by the formula \( \delta_c \leq \delta_2 \delta_1 \). This means that \( \delta_c \) doesn’t exceed \( 5 \cdot 10^{-8} \).

5. Improvement of the phasor discreteness

On the high frequencies the discreteness of the phasor control can be large enough and reach a value of \( 10^{-1} \)–\( 10^{-3} \). However the actual error of the phasor remains low and is much less than the phasor discreteness.

We can reduce the phasor discreteness in different ways. One of them we discuss below.

The multi-bit phasor consists of \( N \) elementary phasor – decades. Fig. 3 shows the structure of such a phasor. Each \( i \)-th decade consists of two DDS\(_{ai}\) and DDS\(_{bi}\). Constant voltage source \( U_{DC} \) supplies every DDS\(_{ai}\) and DDS\(_{bi}\) inputs. The outputs of DDS\(_{ai}\) and DDS\(_{bi}\) are connected to the inputs of the corresponding decade adder \( \sum_i \). The outputs of the adders \( \sum_i \) are connected to the additional inputs of the adders \( \sum_{i+1} \).

These inputs have the weight coefficients \( k_1, k_2, \ldots, k_i, \ldots, k_{n} \). If all decades are the same and have the same absolute \( \Delta U_{ci} \) and relative discreteness \( \theta \), then the coefficients \( k_2, k_3, \ldots, k_n \) must satisfy the conditions:

\[
\prod_{n=2}^{k_i} = \Delta U_{ci} \\Delta U_{cd} \]

In this case, the expression:

\[
U_{\Sigma} = (\alpha + \beta \theta + \ldots + i \theta^{i-1} + \ldots + \gamma \theta_{n-1}) \Delta U d
\]

determines the total voltage of the \( N \)-decade phasor, where:

\( \alpha, \beta, i, \gamma \) – numerical sets in the phasor of the corresponding decade.

\( N \)-decade phasor has essential features due to the nonlinear dependence between the control code and the output voltage of the phasor. For simplicity, we’ll consider here the two-decade phasor.

The output voltage of a decade phasor has to be a continuous function of the summary control code. It means that the range of voltages generated by the phasor of the second decade must be equal to the voltage increment between two neighboring discrete points of the phasor of the first decade.

Equation (17) determines the voltage increment \( \Delta U_{cl} \) of the first decade phasor:

\[
\frac{\Delta U_{cl}}{U_{ml}} = \frac{2 \Delta \Psi cd}{\Delta \Psi_{cd}} \sin i \frac{\Delta \Psi_{cd}}{2} = \frac{1}{2} f_d \quad \frac{\pi}{4} \leq i \frac{\Delta \Psi_{cd}}{f_d} \leq 3 \frac{\pi}{4}.
\]

In accordance with (17), the maximum value \( \Delta U_{cl} \) take place at \( i \frac{\Delta \Psi_{cd}}{f_d} = \frac{\pi}{2} \) and is equal to \( \Delta U_{cl} \max = 2 \Delta \Psi_{cd} \).

To get maximum resolution of two-decades phasor, we’ll assume that the maximum voltage of the phasor of the second decade, acting at the output of the adder \( \Sigma_1 \), is equal to one control sampling of the first decade. Then the equality \( k_2 \cos \frac{\pi}{4} = 2 \sin \frac{\Delta \Psi_{cd}}{2} \) determines the transfer coefficient \( k_2 \) at the third input of the adder \( \Sigma_1 \). This value ensures the continuity of the transfer function of a two-decade phasor only in the vicinity of the point \( i \frac{\Delta \Psi_{cd}}{f_d} = \frac{\pi}{2} \).

When control phase of the first decade increase, the

![Fig. 3. N- decade phasor](image-url)
decrement voltage $\frac{\Delta U_{cd}}{U_{m1}}$ of the first decade decreases. In order to ensure the continuity of the transfer function in the entire voltage range of the first decade, we limit the voltage range of the second decade according to the law:

$$j_{max}\Delta \Psi_{cd} = \arcsin \sqrt{2} \left( \frac{\sin i \Delta \Psi_{cd}}{\sin \Delta \Psi_{cd}} \right),$$

where:

- $j_{max}$ and $\Delta \Psi_{cd}$ are boundary value and the discreteness of the phasor of the second decade.
- With the adopted restrictions, the relative value of the phasor discreteness of the two-decade phasor changes in the entire control range change by $\sqrt{2}$ times, which is insignificant. The differential error of a two-decade phasor does not exceed half of the discreteness unit of the second decade.

### 6. Bridge calibration

Synthesizers accurately change the phase of their output signal. However, the absolute values of the amplitude and phase of their output voltage we know with much less accuracy. To eliminate the proper errors of measurement we calibrate the bridge. This procedure we carry out after the finish of the bridge balancing. It can be done in different ways. Here we consider a method in which each DDS$_a$ and DDS$_b$ synthesizers are calibrated in turn.

To calibrate one of the synthesizers, the zero code is set and held on the second synthesizer, while the first synthesizer generates an output voltage $U_a$ equal to $-U_0$.

The calibration algorithm is based on the variation and the permutation methods. It consists of three stages.

At the first stage, the switch $C_1$ connects the calibration circuit $Z_1Z_2$ between the outputs of the adder $\Sigma$ and the synthesizer DDS$_a$, and VV measures the second value of the calibration bridge output currents.

At the second stage, the MC produces a well-known variation $\delta_a$ of the signal of one of the synthesizers by a value $\Delta U_a$ and VV measures the second value of the unbalance signal $I_{n2}$.

At the third stage, the switch $C_1$ changes the phase of the calibration circuit connection and VV measures the third unbalance signal $I_{n3}$.

Next system of equations describes these procedures:

$$\frac{U_a - U_0}{Z_1} = I_{n1}Z_n = a_{n1} + jb_{n1};$$

$$\frac{U_a + \Delta U_v}{Z_1} - \frac{U_0}{Z_2} = I_{n2}Z_{np} = a_{n2} + jb_{n2};$$

$$\frac{U_a}{Z_2} - \frac{U_0}{Z_1} = I_{n2}Z_{np} = a_{n2} + jb_{n2}.$$

The solution of this system has the form:

$$\delta_a = -1 + \frac{\delta U_a A_i}{2} + \sqrt{1 + \left( \frac{\delta U_a A_i}{2} \right)^2 + \delta U_a A_2} \approx \frac{\delta U_a}{2} \left( A_1 + A_2 \right) + \frac{1}{2} \left( \frac{\delta U_v}{2} \right)^2 \left( A_1^2 - A_2^2 \right),$$

where:

$$\delta_a = \frac{U_a}{U_0} - 1,$$

is error of the DDS$_a$;

$$A_1 = \frac{I_{n1}}{\Delta I_v}, \quad A_2 = \frac{I_{n2}}{\Delta I_v}, \quad \delta U_v = \frac{\Delta I_v Z_{np}}{U_0},$$

$$\Delta I_v = (I_a - I_{n1})$$

is the variation increment of the calibration bridge output currents;

$$\text{rest} \leq \frac{1}{2} \left( \frac{\delta U_v}{2} \right)^3 A_1^2 A_2$$

is the error of the (18) approximation.

The DDS$_a$ synthesizer we calibrate and determine its error $\delta_a = \frac{U_a}{U_0} - 1$ in the same way.

Now, we calculate result of the impedance measurement using the corrected ratio $U_c / U_0$ of signals by formula:

$$U_c / U_0 = \left[ 2 \left( 1 + \frac{\delta_a + \delta_b}{2} \right) \cos 2\Psi \right] e^{-j\Phi}.$$

It should be noted that the DDS output amplitudes do not change during the balancing process. Therefore, the non-linearity of the DDS transfer coefficient does not affect the measurement or calibration error of the bridge. However, the voltage amplitude at the output of the adder $\Sigma$ varies over a wide range. Therefore, the nonlinearity of the adder $\Sigma$ is the part of the measurement error. To increase the linearity of the adder, it can be constructed, for example, using the iterative method [16]. Then the nonlinearity of the transfer function in the audio frequency range can be reduced to values of $10^{-4} - 10^{-7}$.

Experimental studies of the bridge with phase balance showed that in the impedance range of 10.0 Ohm – 10 MOhm, the measurement error doesn’t exceed $(1-5) \cdot 10^{-6}$, and the sensitivity of the bridge is not worse than $(1-2) \cdot 10^{-7}$.

Phase balancing in a reduced form we used in a quadrature bridge to compare the parameters of capacitance and resistance standards [17]. The bridge operates at 1.0 kHz and 1.59 kHz. The range of the impedances measurement is 1.0 kOhm – 100 kOhm.
The bridge comparison error does not exceed $10^{-6}$, and the sensitivity is not worse than $10^{-7}$. In PTB (Germany), we performed the impedance unit transfer from the capacitance $C = 10.0 \, \text{nF}$ to the resistance $R = 10 \, \text{k} \Omega$. This transfer has shown the uncertainty of the $R$ reproduction less than $0.5 \cdot 10^{-6}$, and the uncertainty of dissipation factor $\Delta \text{tg} \delta$ reproduction less than $-15 \cdot 10^{-6}$.

Described above algorithm of the control of the voltage amplitude by adding the cardinal voltages with the controlled phase was proposed in [18, 19]. It should be noted that the interference (adding) of the alternative signals is often used in radio astronomy and radar [20], in ultrasonic flaw detection, ultrasound diagnostics, etc., where, however, high accuracy is not required. For the construction of precision AC bridges, the considered signal control algorithm we proposed for the first time.

7. Conclusions

The bridges with phase control have high accuracy, low dimension and cost. At the limit, such bridges could be developed as integral component. However, one should note that the absolute sensitivity of such bridges is noticeably lower than, for example, of transformer bridges. This limitation arises from the fact that the limiting voltages at the DDS outputs are less than the voltages that can be developed, for example, on the windings of the transformer bridge, and the DDS noise level is noticeably higher than the noise of the transformer bridge.

References