AC phase-controlled bridges – ground and main properties

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Abstract

This work describes the new approach to the improving of the accuracy of the bridge impedance measurements. By this approach, the bridge balance in a wide impedance range uses the **control of the phases of the aggregate of the cardinal signals only**. The optimal structure and algorithm of the control of this aggregate of the cardinal signals is determined. Using this approach accurate impedance meters with low size and price are developed. In the limit, this approach gets us the possibility to develop fully integrated impedance meters. Report shortly describes universal impedance meters with phase control, their balance and calibration algorithms.

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1. Introduction

Impedance measurement has the long history [1]. However, to this day, the creation of precise equipment with high accuracy, small dimensions and weight, does not cease to be relevant.

To determine the parameters of the impedance, many measurement methods have been proposed. Bridge methods get the highest accuracy of impedance measurement.

Historically, four-arm AC bridges were the first used for accurate impedance measurements [1, 2, 3]. In these bridges, to achieve balance, the indicator compares **two output voltages** of two dividers created by the reference standards and the object to be measured. The balancing of bridges is carried out by adjusting one of these voltages using reference resistance and/or capacitance boxes.

In the transformer bridges that appeared later [1, 4, 5, 6], **two voltages** applied to the impedance standard and the measurement object are created using inductive dividers. To balance the bridge, one of the mentioned voltages is adjusted by switching the number of turns of the inductive divider. Such bridges provide measurement with high accuracy, but they are cumbersome and have a relatively narrow frequency range.

Since the late 70s, the accurate digital-to-analog converters (DAC) have been developed. On this base, AC generators were developed [7, 8]. In these generators the DC voltage acts as an input signal, and the output signal is a piecewise approximated sinusoidal voltage (abbreviated – DDS) [8, 9]. DDS were used to create various AC bridges [9, 10]. In these bridges, amplitude and phase of the balancing signal were directly adjusted using DAC. Accuracy of these bridges isn't too much.

In the mid-90s, intensive works were devoted to creation of AC voltage standards based on the Josephson effect [11]. These standards are precision DDS based on either a piecewise linear approximation of a sine waveform or a pulse-driven Josephson array. These works, in the last decade, were used to develop AC bridges [12, 13]. Here **two DDS** based on the Josephson effect act as two arms of the bridge. The standards being compared act as another two bridge arms. To balance such bridges, the amplitude and phase of these DDS are adjusted with high accuracy. Such bridges are potentially very accurate (if the influence of other sources of error inherent in impedance measurement is completely eliminated). However, their complexity, due to superconductivity, is great.

All these AC bridges use for balance adjustment of the amplitude and phase of the proper voltage sources

(or their equivalent). The complexity of such a source determines the technical and economic adjectives of the bridge.

In this work, we propose a new method for adjusting the amplitude of the bridge balancing signal in a wide range of values. This method is based on the adjustment of the phases of the cardinal balancing signals only. Using this approach, we can achieve high accuracy, with small dimensions and weight of the impedance meter.

2. Optimization of the system of cardinal signals

Let us consider the possibility of changing the signal amplitude by changing only the phases of a set of signals.

It is clear that the amplitude of a single signal only can be changed by changing of its actual amplitude only, but not by changing its frequency ω or phase φ . Therefore, consider the sinusoidal voltage U_c , which is the sum of k separate coherent sinusoidal voltages U_i of the same frequency ω with the amplitude U_{mi} and phase φ_i :

$$U_c = \sum_{i=1}^k U_i = \sum_{i=1}^k U_{mi} \sin(\omega t + \varphi_i). \quad (1)$$

Let the amplitudes of the components of the total voltage U_c in (1) be unchangeable constants and cannot be adjusted. Obviously, it is possible to change the amplitude of the total voltage U_c only by changing the phase φ_i of the sinusoidal voltages U_i – the sum component. To create a total voltage with the required amplitude, it is necessary to satisfy only one requirement – the sum of the voltage amplitudes in (1) must be greater than the required total signal amplitude.

The voltages $U_{mi} \sin(\omega t + \varphi_i)$ could be generated using proper DDSs. The phase change error in the DDS is determined only by thermal noise. In the audio frequency domain, the DDS noise can be 10^{-8} or less. Therefore, using DDS, we can balance the bridge by two parameters in a wide range of module and phases of the impedance with high accuracy, changing only the phases φ_i of individual components U_i of the U_c .

2.1. The voltage system, described by the equation (1), does not let us provide an accurate AC bridge. It needs to be optimized. As the optimization criteria, we'll take the dynamic range of the relative change of the U_c and we'll require that this value reach a maximum.

In the process of system (1) optimization it is necessary to answer on the following questions:

– What is the optimal number of the voltage sources used?

– What should be the ratio of these voltages amplitudes?

– What is the optimal law of the voltages phase control?

To answer on these questions let us consider two cases:

- when the number of voltage sources is even;
- when the number of voltage sources is odd.

Let the number k of voltage sources in equation (1) be even.

We'll divide the total sum of k voltage sources by $\frac{k}{2}$ pairs. Then equation (1) takes the form:

$$U_c = \sum_{j=1}^{\frac{k}{2}} U_{mcj} \sin(\omega t + \varphi_{cj}) = \sum_{j=1}^{\frac{k}{2}} [U_{m1j} \sin(\omega t + \varphi_{1j}) + U_{m2j} \sin(\omega t + \varphi_{2j})], \quad (2)$$

where

$U_{m1j}, U_{m2j}, \varphi_{1j}, \varphi_{2j}$ – amplitudes and phases of the j -th pair of voltages.

Formulas (3) determine the amplitude U_{mcj} and phase φ_{cj} of the voltage U_{cj} [14]:

$$U_{mcj} = \sqrt{U_{m1j}^2 + U_{m2j}^2 + 2U_{m1j}U_{m2j} \cos(\varphi_{1j} - \varphi_{2j})};$$

$$\operatorname{tg} \varphi_{cj} = \frac{U_{m1j} \sin \varphi_{1j} + U_{m2j} \sin \varphi_{2j}}{U_{m1j} \cos \varphi_{1j} + U_{m2j} \cos \varphi_{2j}}. \quad (3)$$

Firstly, we find the optimal ratio $r = \frac{U_{m1j}}{U_{m2j}}$ of the amplitudes of the cardinal voltages and determine the algorithm of their phases φ_{1j} and φ_{2j} control ($\varphi_{1j} - \varphi_{2j} = 2\Psi_j$).

Taking into account the above, we transform equations (3) to the form:

$$U_{mcj} = U_{m1j} \sqrt{1 + r^2 + 2r \cos 2\Psi_j}; \quad (4)$$

$$\operatorname{tg} \varphi_{cj} = \frac{\sin \varphi_{1j} + r \sin \varphi_{2j}}{\cos \varphi_{1j} + r \cos \varphi_{2j}}.$$

In the literature, a voltage or current vector is often called a phasor. Here, for brevity, we will extend this term to a device that generates voltage or current according to equations (3), (4).

Equation (4) determines the dependence of the phasor voltage on the ratio r of the amplitudes of the cardinal signals and the difference $2\Psi_j$ in their phases.

We impose two restrictions on these values:

1. The total voltage U_{mcj} is a non-linear function of the phase difference $2\Psi_j$. Therefore, the voltage sensitivity ΔU_{mcj} before changes of the U_{mcj} in the phase difference $2\Psi_j$ is not constant in the dynamic range of phases. Let's define the value ΔU_{mcj} :

$$\Delta U_{mcj} = \left(U_{m1j} \sqrt{1 + r^2 + 2r \cos 2\Psi_j} \right)' \Delta \Psi = \quad (5)$$

$$= -U_{m1j} r \frac{\sin 2\Psi_j}{\sqrt{1 + r^2 + 2r \cos 2\Psi_j}} \Delta \Psi.$$

From (5) we see that the voltage U_{mcj} sensitivity ΔU_{mcj} changes in the control range of the phases Ψ_j according to an approximately sinusoidal law from a relative value of 1 to zero. Obviously, it is impossible to balance the bridge with zero sensitivity. Therefore, we introduce a restriction on the range of angle Ψ_j change: $\frac{\pi}{4} \leq \Psi_j \leq \frac{3\pi}{4}$. Then the control range insignificantly, in $\sqrt{2}$, decreases, and variations in the bridge sensitivity in the control range do not exceed $\sqrt{2}$. Such changes in sensitivity no longer complicate the balancing of the bridge.

2. The voltages U_{mcj} and U_{m1j} are generated by DDS, which have a limited voltage range $U_{\max}$. Therefore, all generated voltages must obey the restrictions: $U_{mcj} \leq U_{\max}$; $U_{m1j} \leq U_{\max}$; $U_{m2j} \leq U_{\max}$ i.e. all arguments in (3), (4) **must be finite** and $0 \leq r \leq r_{\max}$.

From (4), we find the dynamic range D of the phasor, defined as the ratio of the maximum possible voltage U_{mcj} value (at $2\Psi_j = \frac{\pi}{2}$) to its minimum value (at $2\Psi_j = \pi$):

$$D = \frac{(U_{mcj})_{\max}}{(U_{mcj})_{\min}} = \quad (6)$$

$$= \frac{\pm \sqrt{1 + r^2 + 2r \cos \frac{\pi}{2}}}{\pm \sqrt{1 + r^2 - 2r \cos \pi}} = \pm \sqrt{\frac{1 + r^2}{1 + r^2 - 2r}}.$$

From (6) we see that the dynamic range D of the phasor tends to its maximum value (infinity) as r tends to 1. Therefore, we assume that:

$$r = 1 \text{ always.} \quad (7)$$

This equality does not guarantee that the change in the phases of the cardinal voltages will change the amplitude of the output voltage of the phasor only. To determine the law of the phases of cardinal voltages

control, we equate to zero the phase of the output voltage of the phasor, which should not change when the phases of the cardinal voltages change, i.e.:

$$\operatorname{tg} \varphi_{cj} = \frac{\sin \Psi_{1j} + r \sin \Psi_{2j}}{\cos \Psi_{1j} + r \cos \Psi_{2j}} = 0, \quad (8)$$

where:

Ψ_{1j} and Ψ_{2j} – angles between cardinal voltages U_{m1j} , U_{m2j} and voltage U_{mcj} .

From (8) with $r = 1$ we obtain that:

$$\Psi_{1j} = -\Psi_{2j} \text{ always.} \quad (9)$$

Under conditions (7) and (9), the laws of control of the amplitude and phase of the total voltage take the form:

$$U_{mcj} = 2U_{m1j} \cos 2\Psi_j, \quad \Delta \varphi_{cj} = \Delta \varphi_{1j} = \Delta \varphi_{2j}.$$

Fig. 1 demonstrates how the antiphase increment of the phases Ψ_{11} and Ψ_{21} of the cardinal vectors U_{11} and U_{21} to the Ψ_{22} and Ψ_{12} of the cardinal vectors U_{12} and U_{22} changes the amplitude of the summary vector U_c from U_{c1} to U_{c2} , and the sin-phase increment of these angles rotates the acting vector by an angle $\varphi_2 - \varphi_1$.

Fig. 1. Vector diagram of the phase control

Thus, by adjusting only the phases of two cardinal voltages, it is possible to vary within wide limits separately the amplitude and phase of the total voltage used to balance the bridge.

Let now that the phasor contains $\frac{k}{2}$ pairs of DDS, their output voltages are summed up, and the phases of the voltages of each pair are controlled according to the law (10). Then the equations $U_{mcj} = kU_{m1j} \cos 2\Psi_j$, $\Delta \varphi_{cj} = \Delta \varphi_{1j} = \Delta \varphi_{2j}$ describe law of control of the

phasor output voltage, i.e. the amplitude of the output voltage of the phasor increases, in relation to the DDS voltage amplitude, by a factor of k , while maintaining the phase control law.

Let the number k of voltage sources in equation (1) will be odd.

We can select among all k voltage sources such n sources that $n = k - 1$. Then the sum of k voltages can be represented in the following form:

$$U_c = \sum_1^{n/2} (U_{1j} + U_{2j}) + U_k. \quad (11)$$

The first term on the right-hand side of (11) is the sum of the cardinal voltages pairs. The optimization of these pairs we perform as described above. Let us assume that the amplitude and phase of the voltage U_k are constant in time. Then we can consider this voltage as an additive component and easily take it into account in the phasor control algorithm.

2.2. Equation (6) shows that at $r \rightarrow 1$ the width of the dynamic range of the phasor tends to infinity, since in this case the minimum value of the total voltage U_c tends to zero. In practice, however, there are several factors that limit the minimum value of U_c :

1. Not ideal equality of the cardinal voltages: $U_{m1j} \neq U_{m2j}$;
2. Non zero discreteness of the DDS;
3. Non zero level of the noise at the output of the phasor.

Inequality of the cardinal voltages $U_{m1j} \neq U_{m2j}$ leads to a zero shift of the phasor, which is taken into account as a result of calibration.

Phasor discreteness in the simplest case is determined by the ratio of the operating f_0 and clock f_d frequencies of the DDS and can be relatively large. However, we can reduce phasor discreteness to negligible values using multi-decade phasors. This question we will discuss below.

The noise of the DDS components determines the noise at the output of the phasor. This kind of interference can be reduced by the proper choice of components, but in principle, it cannot be reduced to zero.

We need to estimate how DDS noise converts into output noise of the phasor and limits its dynamic range. For this purpose, we assume that in each DDS, in addition to cardinal voltages, there are also some noise voltages $\Delta \bar{U}_{n1}$ and $\Delta \bar{U}_{n2}$. Taking into account equation (3), we can write:

$$U_{mc} + \Delta \bar{U}_{nc} \approx \sqrt{U_{mA}^2 + 2(U_{m1}\Delta \bar{U}_{n1} + U_{m2}\Delta \bar{U}_{n2} + (U_{m1}\Delta \bar{U}_{n2} + U_{m2}\Delta \bar{U}_{n1})\cos 2\Psi)},$$

where:

$$U_{mc}^2 = U_{m1}^2 + U_{m2}^2 + 2U_{m1}U_{m2}\cos 2\Psi; \quad 2\Psi = \Psi_2 - \Psi_1.$$

From the last equation we find the following expression for the signal-to-noise ratio at the phasor output:

$$\frac{\Delta \bar{U}_{nc}}{U_{mc}} = \frac{(r + \cos 2\Psi)\Delta \bar{U}_{n1} + (1 + r\cos 2\Psi)\Delta \bar{U}_{n2}}{U_{m2}(1 + r^2 + 2r\cos 2\Psi)}. \quad (12)$$

Let us assume that the noise $\Delta \bar{U}_{n1}$ and $\Delta \bar{U}_{n2}$ in both channels are not correlated and have the same average value. Usually it is true. Then, using the rules of addition of the random variables, from (12) we obtain:

$$\frac{\Delta \bar{U}_{nc}}{U_{mc}} \approx \frac{\Delta \bar{U}_{n2}}{U_{m2}} \frac{\sqrt{(r + \cos 2\Psi)^2 + (1 + r\cos 2\Psi)^2}}{1 + 2r\cos 2\Psi + r^2}, \quad (13)$$

where:

$$\frac{\Delta \bar{U}_{n2}}{U_{m2}} \text{ and } \frac{\Delta \bar{U}_{nc}}{U_{mc}} \text{ are signal-to-noise ratio at the}$$

DDS and phasor outputs.

From expression (13) we determine how much the signal-to-noise ratio at the output of the phasor has changed in comparison with the signal-to-noise ratio of an individual DDS. Investigation of the equation (13)

showed that the minimum ratio $\frac{\Delta \bar{U}_{nc}}{U_{mc}}$ is achieved at $r = 1$. This ratio is equal to:

$$\frac{\Delta \bar{U}_{nc}}{\Delta \bar{U}_{mA}} = \frac{1}{\sqrt{2}} \frac{\Delta \bar{U}_{n2}}{U_{m2}}. \quad (14)$$

Formula (14) shows that at the output of the phasor the noise-to-signal ratio improves by a factor of $\sqrt{2}$ compared to this ratio at the outputs of the DDS. If $\frac{k}{2}$ DDS pairs are used in the phasor, then the signal-to-noise ratio at the phasor output is improved by $\sqrt{k}$ times. This is a very important property of the phasor.

3. AC bridge with phase-controlled balance

Fig. 2 shows the structure diagram of the AC bridge, balanced by the phasor Φ , with the output voltage U_c controlled in accordance with the laws (12).

Bridge contains synthesizer DDS₀ and phasor Φ with two synthesizers DDS₁ and DDS₂. MC control work of all DDS. Stable DC source U_{DC} supplies the inputs of these DDS. DDS generates the cardinal sinusoidal

Fig. 2. Structure diagram of the phase-controlled AC bridge

voltages U_{11} and U_{21} and reference voltage U_0 . The amplitudes of these voltages have the same value. MC changes their phases in accordance with the algorithm described above. Adder Σ folds the cardinal voltages U_{11} and U_{21} and applies resulting voltage U_c to the measured impedance Z_x . Synthesizer DDS_0 creates the second arm of the bridge. Its output voltage U_0 supplies the impedance Z_0 . Voltmeter VV measures the unbalanced voltage of the bridge at the junction of the impedances Z_x and Z_0 . MC sends the results of VV measurement to the PC. Last one processes them, balances the bridge and forms the measurement result.

4. Bridge balance procedure

To balance the bridge, we will change the phasor output voltage U_c by algorithm described above and use the variation method [15]. In accordance with the variation method, we make the variation $\Delta U_{cv} = U_c \cos \Delta \Psi_v e^{-j\varphi_{c0}}$ of the voltage U_c , where: $\Delta \Psi_v$ – variational increment of angles Ψ_1, Ψ_2 . VV measures the signal of the bridge unbalance before and after the variation. Next system of equations describes these measurements:

$$\frac{U_c}{Z_x} - \frac{U_0}{Z_0} = I_n Z_{np} = a_n + jb_n; \quad (15)$$

$$\frac{U_{c0} + \Delta U_{cv}}{Z_x} - \frac{U_0}{Z_0} = I_{nv} Z_{np} = a_v + jb_v,$$

where:

I_n is the unbalance current of the bridge, a_n, b_n and a_v, b_v are the quadrature components of the VV output voltage before and after the variation, Z_{np} is the VV cross impedance.

We can represent the initial phasor voltage U_{c0} as the sum $U_{c0} = U_{cb} + \Delta U_{c0}$, where U_{cb} is the voltage U_c value at the bridge balance, ΔU_{c0} is the initial distance between the current state of the bridge and its balance point. Taking this into account, from (15) we find ΔU_{c0} :

$$\frac{\Delta U_{c0}}{U_{c0}} = \sqrt{a^2 + b^2} \cdot \cos \Delta \Psi_v e^{-j(\varphi_{c0} + \Delta \varphi_{c0})}, \quad (16)$$

where:

$$a = \frac{a_n(a_v - a_n) + b_n(b_v - b_n)}{(a_v - a_n)^2 + (b_v - b_n)^2},$$

$$b = \frac{b_n(a_v - a_n) - a_n(b_v - b_n)}{(a_v - a_n)^2 + (b_v - b_n)^2}; \quad \Delta \varphi_{c0} = \arctg \frac{b}{a}.$$

We balance the bridge introducing into the phasor the phase increments of the cardinal voltages, calculated according to (16). After that, the voltage U_{c1} , generated by the phasor, will be equal to:

$$U_{c1} = U_{c0} + \Delta U_{c0}.$$

This value of the phasor output voltage is used to calculate the result of the measurement according to the bridge balance equation.

The balance error of the bridge depends on the discreteness of the phasor, the VV discreteness and nonlinearity, noise and interference. Using the modern components, it is easy to construct a VV with a discreteness error of less than 10^{-5} and a nonlinearity of less than 10^{-4} . Then, without taking into account noise, the error δ_1 of the bridge balance don't exceed $2 \cdot 10^{-4}$. The residual bridge unbalance is ΔU_{c1} .

At the second stage of the bridge balance, we reduce the relative value of the variation to a value close to the error δ_1 and, by the same ratio, increase the VV sensitivity. After that, we determine the bridge unbalance ΔU_{c1} . The resulting value ΔU_{c1} we can use in two ways:

- if the discreteness of the phasor control is less than the permissible error of the bridge balance, we enter the control code $U_{c2} = U_{c1} + \Delta U_{c1}$ to the phasor and finish the bridge balance;

- if the discreteness of the phasor control is large, we perform a simple algebraic summation of the voltages U_{c1} and ΔU_{c1} . Using obtained voltage value U_{c2} , we calculate the measurement result.

Excluding noise and interference, the same VV parameters determine the unbalance error δ_2 on the second stage. Therefore, the total error δ_c of

the bridge balance can be estimated by the formula $\delta_c \leq \delta_1 \delta_2$. This means that δ_c doesn't exceed $5 \cdot 10^{-8}$.

5. Improvement of the phasor discreteness

On the high frequencies the discreteness of the phasor control can be large enough and reach a value of 10^{-1} – 10^{-3} . However the actual error of the phasor remains low and is much less than the phasor discreteness.

We can reduce the phasor discreteness in different ways. One of them we discuss below.

The multi-bit phasor consists of N elementary phasor – decades. Fig. 3 shows the structure of such a phasor. Each i -th decade consists of two DDS_{ai} and DDS_{bi}. Constant voltage source U_{DC} supplies every DDS_{ai} and DDS_{bi} inputs. The outputs of DDS_{ai} and DDS_{bi} are connected to the inputs of the corresponding decade adder Σ_i . The outputs of the adders Σ_i is connected to the additional inputs of the adders Σ_{i+1} . These inputs have the weight coefficients $k_1, k_2, \dots, k_i, \dots, k_n$. If all decades are the same and have the same absolute ΔU_d and relative discreteness θ , then the coefficients $k_2, k_i, \dots, k_n$ must satisfy the conditions: $k_1 = k_2 = k_i \dots k_n = \theta$.

In this case, the expression:

$$U_\Sigma = (\alpha + \beta\theta + \dots + i\theta^{i-1} + \dots + \gamma\theta_{n-1})\Delta U_d$$

determines the total voltage of the N -decade phasor, where:

α, β, i, γ – numerical sets in the phasor of the corresponding decade.

N -decade phasor has essential features due to the nonlinear dependence between the control code and

the output voltage of the phasor. For simplicity, we'll consider here the two-decade phasor.

The output voltage of a decade phasor has to be continuous function of the summary control code. It means that the range of voltages generated by the phasor of the second decade must be equal to the voltage increment between two neighboring discrete points of the phasor of the first decade.

Equation (17) determines the voltage increment $\frac{\Delta U_{cli}}{U_{m1}}$ of the first decade phasor.

$$\frac{\Delta U_{cli}}{U_{m1}} = 2\Delta\Psi_{cd} \sin i \Delta\Psi_{cd}; \Delta\Psi_{cd} = \frac{1}{2} \frac{f_0}{f_d}; \quad (17)$$

$$\frac{\pi}{4} \leq i\Delta\Psi_{cd} \leq 3\frac{\pi}{4}.$$

In accordance with (17), the maximum value

$$\frac{\Delta U_{c1max}}{U_{m1}} \text{ take place at } i\Delta\Psi_{cd} = \frac{\pi}{2} \text{ and is equal to } \frac{\Delta U_{c1max}}{U_{m1}} = 2\Delta\Psi_{cd}.$$

To get maximum resolution of two-decades phasor, we'll assume that the maximum voltage of the phasor of the second decade, acting at the output of the adder Σ_1 , is equal to one control sampling of the first

decade. Then the equality $k_2 2 \cos \frac{\pi}{4} = 2 \sin \Delta\Psi_{cd}$ determines the transfer coefficient k_2 at the third input of the adder Σ_1 ($k_2 = \sqrt{2} \sin \Delta\Psi_{cd}$). This value ensures the continuity of the transfer function of a two-decade phasor only in the vicinity of the point $i\Delta\Psi_{cd} = \frac{\pi}{2}$.

When control phase of the first decade increase, the

Fig. 3. N - decade phasor

decrement voltage $\frac{\Delta U_{cl}}{U_{m1}}$ of the first decade decreases.

In order to ensure the continuity of the transfer function in the entire voltage range of the first decade, we limit the voltage range of the second decade according to the law:

$$j_{\max} \Delta \Psi_{cd} = \arcsin \sqrt{2} \left(\frac{\sin i \Delta \Psi_{cd}}{\sin \Delta \Psi_{cd}} \right),$$

where:

$j_{\max}$ and $\Delta \Psi_{2d}$ the boundary value and the discreteness of the phasor of the second decade. With the adopted restrictions, the relative value of the phasor discreteness of the two-decade phasor changes in the entire control range change by $\sqrt{2}$ times, which is insignificant. The differential error of a two-decade phasor does not exceed half of the discreteness unit of the second decade.

6. Bridge calibration

Synthesizers **accurately change** the phase of their output signal. However, the absolute values of the amplitude and phase of their output voltage we know with much less accuracy. To eliminate the proper errors of measurement we calibrate the bridge. This procedure we carry out after the finish of the bridge balancing. It can be done in different ways. Here we consider a method in which each DDS_a and DDS_b synthesizers are calibrated in turn.

To calibrate one of the synthesizers, the zero code is set and held on the second synthesizer, while the first synthesizer generates an output voltage U_a equal to $-U_0$.

The calibration algorithm is based on the variation and the permutation methods. It consists of three stages.

At the first stage, the switch C_1 connects the calibration circuit Z_1-Z_2 between the outputs of the adder Σ and the synthesizer DDS₀, and VV, through the switch C_2 , measures its unbalance signal I_{n1} .

At the second stage, the MC produces a well-known variation δ_v of the signal of one of the synthesizers by a value $\Delta \bar{U}_v$ and VV measures the second value of the unbalance signal I_{nv} .

At the third stage, the switch C_1 changes the phase of the calibration circuit connection and VV measures the third unbalance signal I_{n2} .

Next system of equations describes these procedures:

$$\begin{aligned} \frac{U_a}{Z_1} - \frac{U_0}{Z_2} &= I_{n1} Z_n = a_{n1} + j b_{n1}; \\ \frac{U_a + \Delta U_v}{Z_1} - \frac{U_0}{Z_2} &= I_{nv} Z_{np} = a_{nv} + j b_{nv}; \end{aligned}$$

$$\frac{U_a}{Z_2} - \frac{U_0}{Z_1} = I_{n2} Z_{np} = a_{n2} + j b_{n2}.$$

The solution of this system has the form:

$$\begin{aligned} \delta_a &= -1 + \frac{\delta U_v A_1}{2} + \sqrt{1 + \left(\frac{\delta U_v A_1}{2} \right)^2} + \delta U_v A_2 \approx (18) \\ &\approx \frac{\delta U_v}{2} (A_1 + A_2) + \frac{1}{2} \left(\frac{\delta U_v}{2} \right)^2 (A_1^2 - A_2^2), \end{aligned}$$

where:

$\delta_a = U_a / U_0 - 1$, is error of the DDS_a;

$$A_1 = \frac{I_{n1}}{\Delta I_v}, \quad A_2 = \frac{I_{n2}}{\Delta I_v}, \quad \delta U_v = \frac{\Delta I_v Z_{np}}{U_0},$$

$\Delta I_v = (I_v - I_{n1})$ is the variation increment of the calibration bridge output currents;

$rest \leq \frac{1}{2} \left(\frac{\delta U_v}{2} \right)^3 A_1^2 A_2$ is the error of the (18) approximation.

The DDS_b synthesizer we calibrate and determine its error $\delta_b = U_{1b} / U_0 - 1$ in the same way.

Now, we calculate result of the impedance measurement using the corrected ratio U_c / U_0 of signals by formula:

$$U_c / U_0 = \left[2 \left(1 + \frac{\delta_a + \delta_b}{2} \right) \cos 2\Psi \right] e^{-j\varphi_1}.$$

It should be noted that the DDS output amplitudes do not change during the balancing process. Therefore, the non-linearity of the DDS transfer coefficient does not affect the measurement or calibration error of the bridge. However, the voltage amplitude at the output of the adder Σ varies over a wide range. Therefore, the nonlinearity of the adder Σ is the part of the measurement error. To increase the linearity of the adder, it can be constructed, for example, using the iterative method [16]. Then the nonlinearity of the transfer function in the audio frequency range can be reduced to values of 10^{-6} – 10^{-7} .

Experimental studies of the bridge with phase balance showed that in the impedance range of 10.0 Ohm – 1.0 MOhm, the measurement error doesn't exceed $(1-5) \cdot 10^{-6}$, and the sensitivity of the bridge is not worse than $(1-2) \cdot 10^{-7}$.

Phase balancing in a reduced form we used in a quadrature bridge to compare the parameters of capacitance and resistance standards [17]. The bridge operates at 1.0 kHz and 1.59 kHz. The range of the impedances measurement is 1.0 kOhm – 100 kOhm.

The bridge comparison error does not exceed 10^{-6} , and the sensitivity is not worse than 10^{-7} . In PTB (Germany), we performed the impedance unit transfer from the capacitance $C = 10.0$ nF to the resistance $R = 10$ k Ω . This transfer has shown the uncertainty of the R reproduction less than $0.5 \cdot 10^{-6}$, and the uncertainty of dissipation factor $\Delta \tan \delta$ reproduction less than $-15 \cdot 10^{-6}$.

Described above algorithm of the control of the voltage amplitude by adding the cardinal voltages with the controlled phase was proposed in [18, 19]. It should be noted that the interference (adding) of the alternative signals is often used in radio astronomy and radar [20], in ultrasonic flaw detection, ultrasound diagnostics, etc., where, however, high accuracy is not required. For the construction of precision AC bridges,

the considered signal control algorithm we proposed for the first time.

7. Conclusions

The bridges with phase control have high accuracy, low dimension and cost. At the limit, such bridges could be developed as integral component. However, one should note that the absolute sensitivity of such bridges is noticeably lower than, for example, of transformer bridges. This limitation arises from the fact that the limiting voltages at the DDS outputs are less than the voltages that can be developed, for example, on the windings of the transformer bridge, and the DDS noise level is noticeably higher than the noise of the transformer bridge.

References

- Hall H. P. 2007 *A History of Z measurement*. p. 64. www.ieta-labs.com/pdf/GenRad_History/A_History_of_Z_Measurement.pdf
- Hague B, revised by Foord T.R. 1971 *Alternating Current Bridge Methods*. MA: Pitman. **Sixth Edition**. p. 309.
- Karandeev K.B. 1963 *Special methods of electrical measurement*. [Специальные методы электрических измерений.] Gosenergoizdat p. 344. [In Russian].
- Blumlein A.D. Feb. 1931 *Alternating Bridge circuits*. British Patent No. 323,037. The Illustrated Official Journal of Patents. pp. 7727-772812.
- Awan S., Bryan Kibble B., Schurr J. 2011 *Coaxial Electrical Circuits for Interference-Free Measurements*. The Institution of Engineering and Technology. p. 321. <https://shop.theiet.org/coaxial-elect-circuits>
- Grinevich F. B., Grokholskii A. L., Sobolevskii K. M., Tsapenko M. P. edited by Karandeev K. B. 1970 *Transformer measurement bridges*. [Трансформаторные измерительные мосты.] Energiia. p. 280. [In Russian]
- Murphy E., Slattey C. June 2017 *All About Direct Digital Synthesis*. Ask The Application Engineer. **33**. Version V1.0. <https://www.analog.com/en/analog-dialogue/articles/all-about-direct-digital-synthesis.html#>
- Callegaro L., D'Elia, Bava E., Galzerano, Svelto C., Polyphase 2002 *Synthesizer for Unlike-Impedance Intercomparison System*. CPEM2002. Conference Digest. pp. 176-177.
- A. Corney 2003 *Digital generator assisted impedance bridge*. IEEE Trans. Instrum. Meas. **vol. 52, no. 2**, pp.388 -391.
- Callegaro L. January 2012 *Electrical Impedance: Principles, Measurement, and Applications* Publisher: Taylor & Francis Group. DOI: 10.1201/b13069.
- Overney F., Jeanneret B. 27 July 2018 *Impedance bridges: from Wheatstone to Josephson*. BIPM & IOP Publishing Ltd. Metrologia. **vol. 55, no 5**. DOI: 10.1088/1681-7575/aac6c https://www.researchgate.net/publication/326026114_Impedance_bridges_From_Wheatstone_to_Josephson
- D'Aguilar J. D., Salinas J. R., Kieler O. F., Mendez A. and other. 11 February 2019 *Characterization of an analog to digital converter frequency response by a Josephson Arbitrary Waveform Synthesizer*. IOP Publishing Ltd. Measurement Science and Technology. **vol. 30, no 3**. DOI:10.1088/1361-6501/aafb27 <https://iopscience.iop.org/article/10.1088/1361-6501/aafb27>
- Lee J., Schurr J., Nissila J., Palafox L., Behr R. 2010 *The Josephson two-terminal pair impedance bridge*. Metrologia. **vol. 47, no 4**. DOI:10.1088/0026-1394/47/4/011
- Bronshtein I., Semendyayev K. 1986 *A handbook of mathematics*. [Справочник по математике]. Nauka. p. 544 [In Russian].
- Grinevich F.B., Surdu M.N. 1989 *Highprecision variational AC measurement systems*. [Прецизионные вариационные измерительные системы переменного тока.] Naukova dumka. p. 209. [In Russian]
- Osmolovskiy P. 1969 *Iteration multichannel systems of automated management*. [Итерационные многоканальные системы автоматического управления.] Soviet radio. p. 256 [In Russian]
- Surdu M., Lameko A., Karpov I., Koffman A., Kinard J., Klonz M., Melcher J., Tarlowsky A. 2006 *Complex of bridge comparators for impedance unit traceability and impedance measurements*. Digest of 2006 Conference on Precision Electromagnetic Measurement. Torino, Italy. Digest. pp. 520-521.
- Surdu M., Surdu D. 25.06.2013 *Method of the forming of the voltages with calculated ratios*. [Спосіб формування напруг із заданим відношенням.] Patent of Ukraine 102277. **Bulletin 12** [In Ukrainian].
- Surdu M., Surdu D. August, 2014 *Wide Range Bridge-Comparator for Accurate Impedance Measurements on Audio- and Low Frequencies*. CPEM-2014. Digest. Rio-da-Janeiro. pp. 23-26
- Hansen R. S. 2012 *Phased Array Antennas*. [Фазированные антенные решетки.] **2nd Edition** Technosphera p. 560. [In Russian] http://www.technosphera.ru/files/book_pdf/0/book_267_542.pdf